Abstract

The aim of the present thesis is to investigate the features of Jacobi collocation method for numerical solutions of different types of partial differential equations (PDEs) subject to various kinds of non-local conditions. The speed of convergence is one of the great advantages of spectral collocation method. Moreover, it has exponential rates of convergence; it also has high level of accuracy.

In Chapter 1, we present a general introduction to the spectral methods and their advantages over the standard numerical methods. We also clarify the differences between the three most commonly used spectral methods, namely, the Galerkin, collocation and tau methods. A brief discussion is presented for classifying PDEs and non-local conditions. The orthogonal polynomials, their properties and expansion of functions in terms of them are introduced.

In Chapter 2, we propose two efficient algorithms for solving parabolic PDEs and hyperbolic PDEs in bounded and semi-infinite domains, respectively. A Jacobi Gauss-Lobatto collocation (J-GL-C) method in conjunction with the two stage implicit Runge-Kutta (IRK) scheme are developed for numerical treatment of parabolic PDEs in bounded domain. A new collocation approach is presented to solve hyperbolic PDEs in semi-infinite domain. In this approach, the Jacobi rational Gauss-Radau collocation (JR-GR-C) method is proposed for spatial discretization, with a special choice of the parameters of Jacobi rational functions, and the JR-GR-C method is adopted for temporal discretization with another choice of these parameters. Several illustrative examples are implemented to reveal that the present methods are very effective and convenient for parabolic PDEs and hyperbolic PDEs in bounded and semi-infinite domains.

In Chapter 3, two efficient numerical algorithms are proposed to obtain high accurate numerical solutions for different types of systems of PDEs. In the first one, a Chebyshev-Gauss-Radau collocation (C-GR-C) method in combination with the two stage IRK scheme are employed to obtain highly accurate approximations to the system of nonlinear hyperbolic equations of first

order. In the second algorithm, the J-GL-C method is extended to reduce the nonlinear coupled hyperbolic equations of second order with variable coefficients to a system of algebraic equations, which solved by diagonally-implicit Runge-Kutta-Nyström (DIRKN) method. Finally, the second algorithm is implemented to solve the nonlinear coupled viscous Burgers' equation. Special attention is given to the comparison of the numerical results obtained by the new algorithms with those found by other known methods.

In Chapter 4, we are concerned with the use of Legendre pseudo-spectral approximation in spatial direction to solve numerically parabolic partial differential equations with time-delay. A scalar delay parabolic partial differential equation is then converted into a system of delay differential equations (DDEs) in time direction that can be solved by continuous Runge-Kutta (CRK) scheme. We adapt the algorithm to solve singularly perturbed and coupled time delay parabolic equations. We extend this algorithm to two-dimensional time delay parabolic equations. Some numerical examples are considered to show the effectiveness and accuracy of the present algorithm for solving stiff partial differential equations.

In Chapter 5, we propose an efficient spectral collocation algorithm to solve numerically parabolic and wave type equations subject to initial, boundary and non-local conservation conditions. The shifted Jacobi pseudospectral approximation are investigated for the discretization of the spatial variable of such equations. It possesses the spectral accuracy in the spatial variable. The shifted Jacobi-Gauss-Lobatto (SJ-GL) quadrature rule is established for treating the non-local conservation conditions, and then the problem with its initial and non-local boundary conditions is reduced to a system of ordinary differential equations (ODEs) in temporal variable. This system is solved by two-stage forth-order A-stable IRK scheme. Several numerical examples with comparisons are given. The computational results demonstrate that the proposed algorithm is more accurate than finite difference method, method of lines and spline collocation approach.

Finally in Chapter 6, a J-GL-C method, used in combination with the two stage IRK method, is proposed as a numerical algorithm for the approximation of solutions to nonlinear Schrödinger equations (NLSE) with initial-boundary data in $(1+1)$ dimensions. Our procedure is implemented in two successive steps. In the first one, the J-GL-C method is employed for approximating the functional dependence on the spatial variable, using $(N-1)$ nodes of the Jacobi-Gauss-Lobatto interpolation which depends upon two general Jacobi parameters. The resulting equations together with the two-point boundary conditions induce a system of $2(N-1)$ first-order ODEs in time. In the second step, the implicit Runge-Kutta method of fourth order is applied to solve this temporal system. The proposed J-GL-C method, used in combination with the implicit Runge-Kutta method of fourth order, is employed to obtain highly accurate numerical approximations to four types of NLSE, including the attractive and repulsive NLSE and a Gross-Pitaevskii equation with space-periodic potential. The proposed techinque is extended and developed to solve the complex Schrödinger equations in $(2+1)$ dimensions. The numerical results obtained by these algorithms are compared with various exact solutions in order to demonstrate the accuracy and efficiency of the proposed methods. Indeed, for relatively few nodes used, the absolute error in our numerical solutions is sufficiently small.

The obtained numerical results are tabulated and displayed graphically whenever possible. These results show that our proposed algorithms of solutions are reliable and accurate. Comparisons with previously obtained results by other researchers or exact known solutions are made throughout the context whenever available.

To the best of our knowledge, the formulae and algorithms stated and proved in Chapters 2 up to 6 are completely new. The Programs used in this thesis are performed using the PC machine, with ICPU Intel(R) Core(TM) i3-2350M 2 Duo CPU 2.30 GHz, 6.00 GB of RAM.